BJ and CJ were working on their polar graphing partner quiz. **SCORE: _____ / 40 PTS**

On the question about the polar equation $r = 3 + 2\sqrt{3}\sin 3\theta$, they determined correctly that the symmetry tests $(-r, \theta)$, $(r, -\theta)$, $(-r, \pi - \theta)$ and $(-r, -\theta)$ do **NOT** indicate that the graph is symmetric.

POLE AXIS AXIS $\theta = \frac{\pi}{2}$

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $(r, \pi + \theta)$

(4) $r = 3 + 2\sqrt{3}\sin 3(\pi + \theta)$

$r = 3 + 2\sqrt{3}\sin(3\pi + 3\theta)$

$r = 3 + 2\sqrt{3}(\sin 3\pi \cos 3\theta + \cos 3\pi \sin 3\theta)$ ($\sin 3\pi = 0$, $\cos 3\pi = -1$)

(4) $r = 3 - 2\sqrt{3}\sin 3\theta$

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$

(4) $r = 3 + 2\sqrt{3}\sin 3(\pi - \theta)$

$r = 3 + 2\sqrt{3}(\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta)$ ($\sin 3\pi = 0$, $\cos 3\pi = -1$)

(4) $r = 3 + 2\sqrt{3}\sin 3\theta$

Type of symmetry	Conclusion
Over the pole	CAN'T TELL
Over the polar axis	CAN'T TELL
Over $\theta = \frac{\pi}{2}$	SYM

(4)

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (5)

[c] Find all angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

(5) $3 + 2\sqrt{3}\sin 3\theta = 0$ $\quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(2½) $\sin 3\theta = -\frac{\sqrt{3}}{2}$ $\quad 3\theta \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

(5) $3\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$

(2½) $\theta = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$

AJ throws a football at 20 feet per second, at an angle of 30° with the horizontal, from an initial height of 6 feet. **SCORE: _____ / 15 PTS**
Write parametric equations for the position of the football.

$x = (v_0 \cos\theta)t$

$y = h + (v_0 \sin\theta)t - 16t^2$

$v_0 = 20$

$\theta = 30°$

$h = 6$

(5) $x = (20\cos 30°)t$,

(5) $y = 6 + (20\sin 30°)t - 16t^2$,

$\rightarrow$

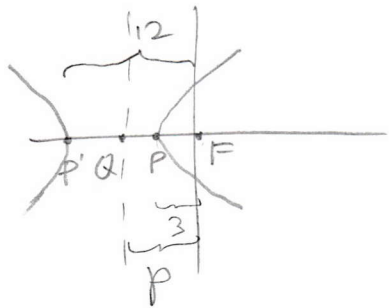
$x = 10\sqrt{3}t$, (2½)

$y = 6 + 10t - 16t^2$, (2½)

A hyperbola has a focus at the pole and vertices with **rectangular** co-ordinates $(-3, 0)$ and $(-12, 0)$. **SCORE: ____ / 25 PTS**

[a] Find polar co-ordinates for the vertices, using positive values of r and θ. **NOTE: You do NOT need to show work.**

(2) $(3, \pi)$, $(12, \pi)$ (2)



[b] Find the **polar** equation of the hyperbola.

(6) $r = \dfrac{ep}{1 - e\cos\theta}$

$e = \dfrac{PF}{PQ} = \dfrac{P'F}{P'Q}$

$r = \dfrac{\frac{5}{3} \cdot \frac{24}{5}}{1 - \frac{5}{3}\cos\theta} \cdot \dfrac{3}{3}$ (3)

$e = \dfrac{3}{p-3} = \dfrac{12}{12-p}$ (5)

$36 - 3p = 12p - 36$

$72 = 15p$

$p = \dfrac{24}{5}$ (3)

(2) $r = \dfrac{24}{3 - 5\cos\theta}$

(2) $e = \dfrac{3}{\frac{24}{5} - 3} \cdot \dfrac{5}{5} = \dfrac{15}{24 - 15} = \dfrac{15}{9} = \dfrac{5}{3}$

Find the logarithmic formula for $\tanh^{-1} x$ by solving $x = \tanh y$ for y **SCORE: _____ / 25 PTS**

using the exponential definition and an algebraic substitution $z = e^y$ (or a similar substitution).

(5) $x = \dfrac{e^y - e^{-y}}{e^y + e^{-y}} = \dfrac{z - \frac{1}{z}}{z + \frac{1}{z}} \cdot \dfrac{z}{z} = \dfrac{z^2 - 1}{z^2 + 1}$ (3)

(3) $xz^2 + x = z^2 - 1$

$xz^2 - z^2 = -x - 1$

(6) $z^2(x-1) = -x - 1$

(3) $z^2 = \dfrac{-x-1}{x-1} = \dfrac{1+x}{1-x}$

$e^{2y} = \dfrac{1+x}{1-x}$

$2y = \ln \dfrac{1+x}{1-x}$

$y = \left|\frac{1}{2}\right| \ln \dfrac{1+x}{1-x} = \tanh^{-1} x$

(2) (3)

Name the shape of the graphs of the following polar equations. Be as specific as possible.
If the graph is a rose curve, state the number of petals.

SCORE: _____ / 20 PTS

[a] $r = \dfrac{5}{2+3\cos\theta}$ (3) HYPERBOLA

[b] $r = 5+2\cos\theta$ (3) CONVEX LIMACON

[c] $r = 3-5\sin\theta$ (4) LIMACON WITH LOOP

[d] $r = 3\cos\theta$ (3) CIRCLE

[e] $r = 3\sin 2\theta$ (4) ROSECURVE (4 PETALS)

[f] $r = \dfrac{5}{3-2\sin\theta}$ (3) ELLIPSE

Eliminate the parameter for the parametric equations $x = \frac{t}{3-t}$, $y = \frac{t-1}{2+t}$.

SCORE: _____ / 15 PTS

Write your final answer in the form $y = f(x)$ completely simplified.

$3x - xt = t$

$3x = t + xt$ (4)

$3x = t(1+x)$

$t = \frac{3x}{1+x}$ (3)

(4) $y = \frac{\frac{3x}{1+x} - 1}{2 + \frac{3x}{1+x}} \cdot \frac{1+x}{1+x}$

$y = \frac{3x - (1+x)}{2(1+x) + 3x}$

(4) $y = \frac{2x-1}{2+5x}$

Rewrite $\coth\left(-\frac{1}{2}\ln x\right)$ in terms of exponential functions and simplify.

SCORE: _____ / 10 PTS

(3) $\dfrac{e^{-\frac{1}{2}\ln x} + e^{\frac{1}{2}\ln x}}{e^{-\frac{1}{2}\ln x} - e^{\frac{1}{2}\ln x}}$

$= $ (4) $\dfrac{\frac{1}{\sqrt{x}} + \sqrt{x}}{\frac{1}{\sqrt{x}} - \sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}}$

$= $ (3) $\dfrac{1+x}{1-x}$

OR

(3) $\dfrac{e^{-\ln x} + 1}{e^{-\ln x} - 1}$

$= $ (4) $\dfrac{\frac{1}{x} + 1}{\frac{1}{x} - 1} \cdot \dfrac{x}{x}$

$= $ (3) $\dfrac{1+x}{1-x}$